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The velocity required for the impact destruction of small particles is determined. It is established that energy is exchanged between particles accelerated in a gas stream. The particle density distribution when two dust-laden streams meet is found.

In counterflow jet pulverizers pulverization results mainly from the collision of opposing streams of particles accelerated in acceleration tubes by a gas flow. The theory of this process is still far from complete [1]. Below some approximate solutions to certain problems in the theory of jet pulverization are discussed.

Let us examine the question of the velocity that the particles must acquire to have sufficient kinetic energy for destruction on collision with a wall or particles of the same or larger mass.

According to [5], the critical velocity for the case of collision of a sphere with a rigid wall is determined by the elastic constants and strength of the solids, but not by their dimensions. This conclusion, based on the theory of Hertz, is valid only for absolutely brittle bodies, for which the elastic strains remain proportional to the stress right up to fracture. It is known, however, that even such brittle minerals as quartz, calcite, and diamond undergo plastic deformation before fracturing [6]. It has been shown that the energy of limiting plastic deformation (and that expended on the creation of new surfaces) is inversely proportional to the size of the disintegrating particles, and may considerably exceed the energy of limiting elastic deformation [3].

Let us examine how the energy of limiting plastic deformation and the energy of formation of new surfaces affect the critical velocity.

Using the Hertz theory on the relation between the force produced by the head-on collision of two identical spheres and the distance between their centers [7, 8] and assuming, as in [5], that a brittle spherical particle disintegrates when the stresses at the maximum section become equal to the elastic limit, we find the energy of limiting elastic deformation:

$$\varepsilon_1 = \frac{\pi}{12} \delta^3 \sqrt{P^5/E^2}. \quad (1)$$

It is known that plastic deformation of brittle solids is concentrated in layers adjacent to the surfaces, along which disintegration takes place [3, 9, 10]. The depth of the limiting plastic deformations of brittle solids is small – not more than 100-150 Å for quartz at ordinary temperatures [3, 4, 6]. Therefore the energy of plastic deformation of a particle can be defined as:

$$\varepsilon_2 = a \beta l \delta^2. \quad (2)$$

On disintegration the energy expended on increasing the free surface of the particle is equal to

$$\varepsilon_3 = a \sigma \delta^2. \quad (3)$$

Since it has been shown experimentally that the energy expended during impact pulverization and static crushing is approximately the same [11, 13], to determine the value of the critical velocity, W_d , we may equate the kinetic energy of the flying particle, taking into account the efficiency of the process (i. e., unavoidable losses due to friction, noise, etc.), to the energy required for its destruction:

$$\begin{aligned} \frac{mW_d^2}{2} \eta &= \frac{\pi\delta^3}{12} \rho W_d^2 \eta = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \\ &= \frac{\pi}{12} \delta^3 \sqrt{\frac{P^5}{E^2}} + a \beta l \delta^2 + a \sigma \delta^2. \end{aligned} \quad (4)$$

Solving (4) for W_d and δ , we obtain

$$W_d = \left\{ \frac{1}{\rho\eta} \left[\sqrt[3]{\frac{P^5}{E^2}} + \frac{12a(\beta l + \sigma)}{\pi\delta} \right] \right\}^{1/2}; \delta_{\min} = \frac{12a(\beta l + \sigma)}{\pi(W_d^2 \rho\eta - \sqrt[3]{P^5/E^2})}. \quad (5)$$

From (5) it follows, in particular, that, for relatively large particles of brittle material of the quartz type, when the energy expended on irreversible deformation and on increasing the free surface may be neglected in comparison with the energy of limiting elastic stress, W_d will cease to depend on particle size.

The actual calculation of W_d is complicated by the need to determine the depth l of the plastic strains, which depends on the temperature, the material properties, and the manner of application of the load. It is no less difficult, and sometimes impossible, to evaluate the energy density of the limiting plastic strain. The process of pulverization of quartz is most favorable in this respect; here l has been evaluated from the thickness of the amorphous layer, and the lower limit of the energy density of limiting plastic deformation from the crystallization energy [3]. It has been determined experimentally that for dry pulverization of quartz sand at normal temperatures $l = 1.5 \cdot 10^{-8}$ m. The lower limit for β has been found to be $\beta = 4.4 \cdot 10^8$ J/m³, $\sigma = 1.0$ J/m², $a = 3$, $\eta = 0.9$, $P = 2.75 \cdot 10^8$ N/m², $E = 9.8 \cdot 10^{10}$ N/m², $\rho = 2.5$ g/cm³.

In particular, it follows from these data that the energy expended on plastic deformation, $\beta l = 6.6$ J/m³, is considerably greater than that expended on forming new surfaces.

The results of calculating the critical velocity with the above values for the parameters are presented in Fig. 1,

from which it may be seen that this velocity increases rapidly as the size of the particles diminishes.

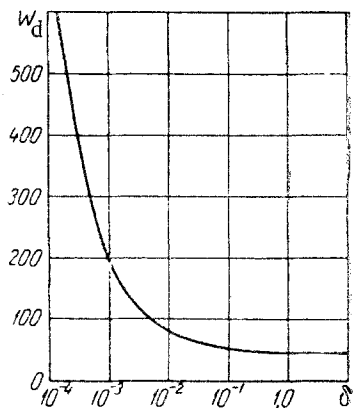


Fig. 1. Dependence of critical velocity (W_d , m/sec) on diameter of quartz particles (δ , mm).

It is interesting to examine how far the velocity of the particles at the nozzle exit of the acceleration tube depends on their size.

Integrating the equation of motion of particles accelerated by a gas stream with constant velocity and density [5], we obtain the particle velocity

$$W = c [1 - \delta\rho / (0.36 \gamma c \tau + \delta\rho)]. \quad (6)$$

It follows from (6) that the smaller the particle, the greater the velocity it acquires in the same time interval. Small particles will therefore overtake larger particles, and, if there is a collision, push them along. This situation is illustrated graphically in Fig. 2, in which the velocity of particles of various sizes presented as a function of acceleration time.

The possibility of energy transfer between particles during acceleration is determined by the probability of collision along the length of the acceleration tube. The considerable concentration of highly dispersed solid particles in the two-phase flow and the randomness of their motion, due to the substantial difference in the velocities of particles of different sizes in a coordinate system moving at the mean flow velocity, allow such a system to be regarded as a gas moving under the action of a certain pressure. This analogy is confirmed by an evaluation of the mean free path of particles in a dust-laden flow. In fact, in a coordinate system moving at the mean particle flow velocity, the mean free path λ in a system of N monodisperse particles may be written

$$\lambda = 4/\pi \sqrt{2} \delta^2 N = 0.47 \delta\rho/\mu. \quad (7)$$

Values of λ calculated for a number of discrete values of δ and μ with $\rho = 2.5$ g/cm³ are:

μ , N/m ³	10			30			100			150		
δ , mm	1	0.1	0.001	1	0.1	0.01	1	0.1	0.001	1	0.1	0.01
λ , mm	1170	117.0	11.700	390	39.0	3.90	117	11.7	1.170	78	7.8	0.78

The length of the acceleration tubes in jet pulverizers varies between 170 and 500 mm at diameters of from 30 to 60 mm. The mean free time for particles with $\delta = 0.1$ mm does not exceed 10^{-4} sec.

It follows from the data and Fig. 2 that, for a solids content of the order of 30-100 N/m³ and particle sizes of the order of 0.1 mm and less, the particles will in fact behave, in a certain sense, like gas molecules, and on their way through the acceleration tube will undergo a considerable number of collisions with one another, exchange momentum, and exit with approximately the same velocity irrespective of size.

This velocity may be estimated from (6) for the average particle size. In doing so, however, it is necessary to take into account that experimental data show that the particle velocity is only 0.4-0.8 of the flow velocity [2]. It would therefore be more correct to replace the flow velocity in (6) with the limiting particle velocity.

Let us suppose that particles of one stream penetrate freely into a counter stream of particles, without suffering deceleration due to the gas.

This hypothesis is valid under certain limited conditions: when, firstly, the gas velocities in the region where the streams meet are not large and not directly opposed to the particle motion, which is the case in counter-flow jet pulverizers of normal construction, and, secondly, when the particles succeed in penetrating a small distance into the opposing stream.

Let us suppose further that the particles are carried out of the impact-region after collision. This is true when the width of the impact region is small and the drift velocity is substantial, which is the case in practice.

From these assumptions it follows that all the particles collide, and half of them reach the plane in which the streams meet, since the streams are symmetrical, $n_0 = N/2$.

The total number of colliding particles at the meeting point, coming from both streams, is N , and this remains true, with the above assumptions, for any section of the jet. Indeed, let n particles reach any section of the jet. These n particles are capable of knocking out only n opposing particles, and so the total number of particles reaching the chosen section is

$$N_1 = N - n + n = N. \quad (8)$$

Let us determine the dependence of the density of the number of collisions on distance from the plane of symmetry.

When the streams meet, the probability of collision is determined by the cross-sectional area of the particles in unit cross section of the flow. The number of particles dn undergoing collision over a path length dh is

$$-dn = dh(N - n)\pi\delta^2 n. \quad (9)$$

The quantity $dh(N - n)$ in (9) is the number of particles in a layer of thickness dh and cross-sectional area 1 cm^2 , perpendicular to the flow. The effective cross-sectional area $\pi\delta^2$ is taken with allowance for the size of both the colliding particles.

Integrating (9) and solving it for n and h , we obtain

$$\frac{n}{N} = \frac{1}{\exp(\pi\delta^2 hN) + 1} = \frac{1}{\exp(6\mu h/\rho\delta) + 1}. \quad (10)$$

From (10) we find the distance in which only one percent of the particles has not collided, i. e., $n/N = 1/100$. Then, evidently,

$$h = \rho\delta \ln 99/6\mu = 0.765 \rho\delta/\mu. \quad (11)$$

The total distance in which 99% of the particles have collided is

$$H = 2h = 1.53 \rho\delta/\mu. \quad (11a)$$

Figure 3 shows the distribution of collision density for particles of various sizes at a solids content of the order of 50 N/m^3 as a function of distance from the midpoint between the ends of the acceleration tubes (the meeting point of the streams). It will be seen from Fig. 3 that practically all particles 100μ or less in size collide in a distance of less than 50 mm from the meeting point of the streams. For 250μ particles, this distance is 100 mm .

It should be noted that deceleration due to the gas modifies the probability of collision to some extent, as Fig. 3 illustrates, since particles which have passed without colliding beyond the meeting plane to the section of the opposing acceleration tube may lose velocity and be carried out of the collision zone.

Experimental investigations [1] of the jet pulverization of quartz sand at a solids content 50 N/m^3 , mean particle diameter 0.5 mm , and distance between ends of acceleration tubes 120 mm have shown that $\sim 50\%$ of the particles collide.

Figure 3 shows that $\sim 60\%$ of particles of diameter 0.5 mm collide in a zone of length 120 mm , which to some extent confirms the correctness of the assumptions underlying the calculations.

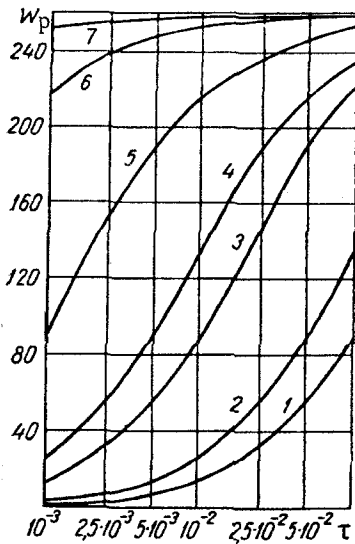


Fig. 2. Velocity (W_p , m/sec) of a particle accelerated by a gas stream as a function of acceleration time (τ , sec): 1) particle diameter 10 mm ; 2) 5 ; 3) 1 ; 4) 0.5 ; 5) 0.1 ; 6) 0.01 ; 7) 0.001 .

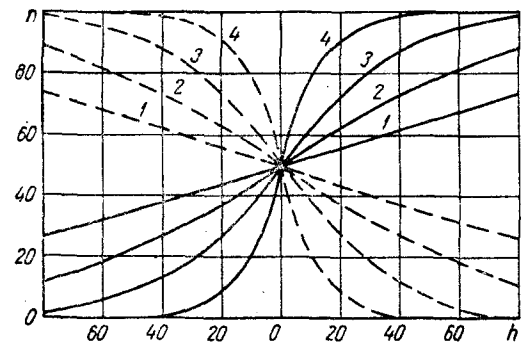


Fig. 3. Number of colliding particles (%) as a function of distance (h , mm) from the meeting plane of the two-phase flows for $\mu = 50 \text{ N/m}^3$; continuous line - particle flow from left to right; broken line - right to left: 1) particle diameter 1 mm ; 2) 0.5 ; 3) 0.25 ; 4) 0.1 .

NOTATION

δ - particle diameter; E, ρ, m, P - Young's modulus, density, mass, and elastic limit of particle material; l - depth of limiting plastic strains; a - increase in particle surface for single fracture; ϵ_1, ϵ_2 - energy of limiting and plastic deformation; ϵ_3 - energy of increase in free surface; β - energy density of plastic deformation; σ - free energy of unit surface of particle material; η - efficiency of fracture; W_d - critical velocity; W - particle velocity; c, γ - velocity and density of gas; τ - time; h - path length; λ - mean free path of particle; N, μ - number and total mass of particles in unit volume; n - number of particles in a given section; n_0 - number of particles reaching plane of symmetry of opposing streams.

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